BOUNDARY LAYERS IN A FULLY IONISED TWO-TEMPERATURE PLASMA

(POGRANICHNYE SLOI V POLNOST'IU IONIZOVANNOI DVUKHTEMPERATURNOI PLAZME)

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In the following article boundary layer equations are deduced for a fully ionised two-temperature plasma. It is assumed that the plasma can be described by the equations developed in [1], incorporating the simplifications of [2]. If we study the behavior of the temperature close to the walls in a two-temperature plasma, we are faced with two temperature boundary layers, namely, an electronic boundary layer (where there are sharp changes in electron temperatures) and an ionic boundary layer (where ionic temperatures evince sharp change). Equations are derived for the electronic and ionic boundary layers. It is established that the thickness of the electronic boundary layer is much greater than that of the ionic temperature boundary layer. An approximate method is given for evaluation of boundary layers in two-temperature plasmas. Cases in which electronic temperature boundary layer is absent, are given.

Apart from the existence of boundary layers exhibiting sharp temperature changes in electrons and ions, it is also shown that a two-temperature conducting plasma can embody a specific 'screening', boundary layer adjacent to the wall, in which any temperature difference of electrons and ions caused by outside sources close to the wall culminates in values determinable by the combined action of viscosity, Joule heat generation and heat conductivity- consistent with the afore-mentioned equations. The thickness of this layer $\delta \sim \sqrt{\varkappa_i / \gamma}$ is, over a wide range of defining parameters, much less than that of the dynamic or the ionic boundary layer.

The behavior of the electron and ion temperatures within the layer is investigated. We also show that a thin 'screening' layer of thickness $l \sim L / \gamma^\circ$, can be established in the external stream, when the viscosity and the thermal conductivity of the medium can be neglected. It is in this layer, that sharp changes in electron and ion temperature differences from those caused by external sources at some sections of the stream, e.g. at the entry to the channel, up to those determined by Joule heating in accordance with the equations for the external stream, are set up. These layers are the characteristic feature of two-temperature plasma, in single-temperature plasma they do not exist.

Notation.

We shall use the suffix e to denote quantities relating to electrons, i to ions. Index $^\circ$ denotes dimensioness quantities, w denotes the values near the wall, whilst δ defines the values at the distance δ from the wall and ∞ denotes values outside the stream at infinity.

n is the number of particles in unit volume:

ρ is the density of mixture;

m is the mass of a particle;

$\mathbf{v}_e$ and $\mathbf{v}_i$ are the mean velocities of electron and ion components;

$\mathbf{v}\,(v_x v_y)$ is the mean velocity of mixture;

ω is the cyclotron frequency;

c is the velocity of light;

p, and π are the pressure and the tensor of viscous stresses;

$\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields;

$\mathbf{j}$ is the current density;

T_e and T_i are the electron and ion temperatures;

$\mathbf{q}$ is the heat flux;

γ is the coefficient preceding temperature difference between electrons and ions in the energy equations;

τ_e and τ_i is the time interval between collisions of electrons and ions respectively.

$N, L. T$ and U is the characteristic number of particles per unit volume, length, temperature and velocity of the medium respectively;

$\eta_{e,i}$, $\varkappa_{e,i}$, and σ are the coefficients of viscosity, thermal conductivity and the conductance of the medium;

δ_v, δ_e, and δ_i are the thicknesses of the viscous, the electron-temperature and ionic-temperature magnetohydrodynamic boundary layers

δ is the thickness of the δ-layer in which the temperature differences between the electrons and ions within the thermally conducting plasma change sharply from the values determined by external sources close to the wall to the values determined by the given equations.

l is the thickness of the γ-layer in which the temperature difference between the electrons and ions undergoes a sharp change from the values determined by external sources at the channel entry, to the values determinable from Joule heat in accordance with the equations written down for the external stream.

$$\xi = \frac{x}{L}, \quad \eta = \frac{y}{L}, \quad u\,(v) = \frac{v_x\,(v_y)}{U}, \quad \theta_j = \frac{T_j}{T}, \quad j_0 = \frac{j}{\sigma U H / c}$$

$$\lambda = \frac{mU^2}{kT}, \quad \gamma^\circ = \frac{\gamma L}{NU}, \quad \varkappa^\circ = \frac{\varkappa}{NUL}, \quad R = \frac{\rho UL}{\eta_i}, \quad R^\circ = \frac{1}{\varkappa^\circ_i} = 0.246R$$

$$M^2 = \frac{H^2 \sigma L^2}{c^2 \eta_i}, \quad \varphi^2 = \gamma^\circ R^\circ \left(1 + \frac{\varkappa_i}{\varkappa_e}\right) \equiv \gamma L^2 \left(\frac{1}{\varkappa_e} + \frac{1}{\varkappa_i}\right) \approx \gamma^\circ R^\circ \equiv \frac{\gamma L^2}{\varkappa_i}$$

1. Fundamental equations and the dynamic boundary layer. The events which occur in a fully ionised plasma will be described by the equations obtained in [1], using simplifications from [2]. For clarity we will deal with the case where $\omega_i\tau_i \lesssim \omega_e\tau_e \ll 1$ (extension to the case of $\omega_e\tau_e \sim 1$ is easy). Expressions for ω_e, ω_i, τ_e and τ_i are derived in [1]. Note, that to fulfill the first inequality it is essential that the ion temperature T_i should not be more than $(m_i/2m_e)^{1/3}$ times greater than the electron temperature, T_e. The second inequality [3] is fulfilled under the conditions close to the real ones ($H = 10^4$ gauss, $T_e = 1$ ev.) when the number of electrons $n_e \gtrsim 10^{16}$ cm^{-3}.

The equations which describe the behavior of a fully ionised, two-temperature, quasi-neutral ($n_e \approx n_i = n$), plasma, under the given assumptions become [1 and 2]

$$\frac{\partial\rho}{\partial t}+\operatorname{div}\rho\mathbf{v}=0 \tag{1.1}$$

$$\frac{\partial\rho v_\alpha}{\partial t}=-\frac{\partial p}{\partial x_\alpha}-\operatorname{div}\rho\mathbf{v}v_\alpha-\frac{\partial\pi^{\alpha\beta}}{\partial x_\beta}+\frac{1}{c}[\mathbf{j}\times\mathbf{H}]_\alpha \tag{1.2}$$

$$\frac{3}{2}n_e\frac{d_eT_e}{dt}+p_e\operatorname{div}\mathbf{v}_e=-\operatorname{div}\mathbf{q}_e-\pi_e^{\alpha\beta}\frac{\partial v_{e\alpha}}{\partial x_\beta}+Q_e \tag{1.3}$$

$$\frac{3}{2}n_i\frac{d_iT_i}{dt}+p_i\operatorname{div}\mathbf{v}_i=-\operatorname{div}\mathbf{q}_i-\pi_i^{\alpha\beta}\frac{\partial v_{i\alpha}}{\partial x_\beta}+Q_i \tag{1.4}$$

$$\mathbf{j}=\sigma\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{H}\right) \tag{1.5}$$

$$\pi^{\alpha\beta}\approx\pi_i^{\alpha\beta}=-\eta_i\left(\frac{\partial v_{i\alpha}}{\partial x_\beta}+\frac{\partial v_{i\beta}}{\partial x_\alpha}-\frac{2}{3}\delta_{\alpha\beta}\frac{\partial v_{il}}{\partial x_l}\right)\qquad(k=ei)$$

$$\pi_k^{\alpha\beta}\frac{\partial v_{k\alpha}}{\partial x_\beta}=-\eta_k\left\{2\left[\left(\frac{\partial v_{kx}}{\partial x}\right)^2+\left(\frac{\partial v_{ky}}{\partial y}\right)^2\right]+\left(\frac{\partial v_{ky}}{\partial x}+\frac{\partial v_{kx}}{\partial y}\right)^2-\frac{2}{3}\left(\frac{\partial v_{kx}}{\partial x}+\frac{\partial v_{ky}}{\partial y}\right)^2\right\}$$

$$Q_e=\frac{j^2}{\sigma}+0.711n_e(\mathbf{v}_e-\mathbf{v}_i)\nabla T_e-\gamma(T_e-T_i),\quad Q_i=\gamma(T_e-T_i),\quad \gamma=\frac{3m_en_e}{m_i\tau_e}$$

$$\mathbf{q}_e=0.711\,(\mathbf{v}_e-\mathbf{v}_i)\,n_eT_e-\varkappa_e\nabla T_e,\quad \mathbf{q}_i=-\varkappa_i\nabla T_i,\quad \mathbf{j}=en(\mathbf{v}_i-\mathbf{v}_e) \tag{1.6}$$

$$\sigma=\frac{1}{0.5129}\frac{e^2n\tau_e}{m_e},\quad \varkappa_e=3.1616\frac{n_eT_e\tau_e}{m_e},\quad \varkappa_i=3.906\frac{n_iT_i\tau_i}{m_i},\quad \eta_i=0.96\,n_iT_i\tau_i$$

In order to complete the system, equations of state for both electrons and ions should be added $p_e=n_ekT_e$, $p_i=n_ikT_i$ together with Maxwell's equations (List of symbols is given at the beginning of the article).

It is easy to check the validity of the following identity

$$\frac{\mathbf{j}}{enU}=\omega_e\tau_e\frac{\mathbf{j}}{0.5129\,\sigma\,UH/c} \tag{1.7}$$

In the equations of motion we have ommitted the electron viscosity. This can be done [2] when $T_e\ll(2m_i/m_e)^{1/3}T_i$ for $j/enU\lesssim 1$. This criterion should be used, since $\omega_e\tau_e\ll 1$, and by virtue of (1.7). Apart from that the term $e(n_i-n_e)\mathbf{E}$, has been left out of the equation of motion and out of the Ohm's Law $e(n_i-n_e)\mathbf{v}$. It is well known that these terms can be neglected in magnetohydrodynamics and it is easy to show that the same applies in case of the equations of magnetohydrodynamic boundary layers. For the case where the parameter of mutual influence $M^2/R\sim 1$, appropriate estimates were obtained in [4].

Let us assume, for the sake of clarity, that the mutual influence parameter $M^2/R\sim 1$, whilst the inertia terms are of the same order as the electromagnetic terms. Suppose that a velocity change of the order of the velocity itself, takes place within a layer of thickness L. Also, suppose that the order of magnitude of the inertia term is $\rho U^2/L$, that of the viscous term is $\eta U/L^2$, while that of the electromagnetic term is $jH/c\sim\sigma UH^2/c^2$. If we now compare the orders of magnitude of the terms in the equations of motion (1.2), we arrive at

$$\frac{\text{electromag.}}{\text{inertial}} \sim \frac{M^2}{R} \sim 1, \quad \frac{\text{electromagn.}}{\text{viscous}} \sim M^2, \quad \frac{\text{inertial}}{\text{viscous}} \sim R \tag{1.8}$$

It is easy to see that when $M^2 \sim 1$, the Reynolds number $R \sim 1$, and the equation of motion has the most general form (1.2). When $M^2 \gg 1$ the Reynolds No. is also $R \gg 1$; viscous terms can be neglected in the main flow. Let us assume that there is a thin layer of thickness δ_v, in which the change of velocity is comparable with its value. In this case the order of magnitude of the viscous terms is equal to $\eta_i U / \delta_v^2$, while the order of the inertial and electromagnetic terms remains as before $\sigma U H^2 / c^2$.

It is easy to see that the viscous terms can be of the same order of magnitude as the electromagnetic ones only within the thin boundary layers of thickness $\delta_v \sim L / \sqrt{R}$. Outside of the boundary layers, the viscous terms can be neglected. It is also apparent that when $M^2 / R \sim 1$ the thickness of the viscous magnetohydrodynamic boundary layer coincide with that of the conventional viscous hydrodynamic boundary layer.

Such boundary layers, which in the following we shall refer to as the viscous magnetohydrodynamic boundary layers were investigated in [4 to 6] in the case of single temperature plasma.

Note that when $T_e/T_i \sim (2m_i / m_e)^{1/5}$, then $\pi_e \sim \pi_i$ [2], and in this case the electron viscosity should be taken into account in the equations together with the ionic viscosity. When $T_e/T_i \gg (2m_i/m_e)^{1/5}$, then $\pi_e \gg \pi_i$, and electron viscosity should be retained in the equations, while the ionic viscosity can be neglected. Also in the above estimates the quantity $R_e = \rho U L / \eta_e$ should replace R. In this case the thickness of the viscous boundary layer will be

$$\delta_v \sim L / \sqrt{R_e}$$

Note that the geometry of the flow can be such, that the projection of the electromagnetic terms onto the normal is equal to zero, and therefore $\partial p / \partial y = 0$, as in conventional gasdynamics. In general however, $\partial p / \partial y \neq 0$.

2. Boundary layer for the difference of temperatures of electrons and ions in a quiescent plasma. To clarify the behavior of the temperatures of electrons and ions close to the wall, we shall first consider a special case which is interesting in itself. We shall assume that in Equations (1.3) and (1.4) the terms describing convective heat transfer, the work of viscous forces and the changes in temperature in the direction of the x-axis can be neglected. To make things definite we will assume $\rho =$ const. Then, equations (1.3) and (1.4). taking account of (1.7), become

$$\frac{d}{d\eta}\frac{\varkappa_e}{\varkappa_i R^\circ}\frac{d\theta_e}{d\eta} + \frac{M^2}{R}\lambda j_0^2 - \gamma^\circ(\theta_e - \theta_i) = 0 \tag{2.1}$$

$$\frac{d}{d\eta}\frac{1}{R^\circ}\frac{d\theta_i}{d\eta} + \gamma^\circ(\theta_e - \theta_i) = 0 \tag{2.2}$$

It should be emphasised that the characteristic velocity U, and with it, the Reynolds No., have been used in deriving (2.1) and (2.2). As a result of this, the equations (2.1) and (2.2) can be regarded as a special case of the general energy equations (1.3) and (1.4), written in a nc -dimensional form. It is these equations that are used to describe the heat transfer in fully ionised, quiescent, two-temperature plasma in a onedimensional framework. It sho ld be noted that the assumption of onedimensionality does not affect the generality of the results which follow (see also the remarks at the end of this section). Heat transfer during the motion of a fully ionised two-temperature plasma in a plane channel in presence of a magnetic field [3] can also be represented by Equations (2.1) and (2.2), but in this case a term $\lambda (\partial u / \partial \eta)^2 / R$, representing work against ionic viscosity, must be added.

It must be emphasised that on the surface of the wall the electron temperature can differ from the ion temperature. For simplicity let us assume coefficients $\varkappa_e$, $\varkappa_i$, and R to be constant. Subtracting (2.2) multiplied by R° from Equation (2.1) multiplied by $\varkappa_i R^\circ / \varkappa_e$, we obtain the equation for the temperature differences between the electrons and ions

$$\frac{d^2(\theta_e - \theta_i)}{d\eta^2} - \varphi^2(\theta_e - \theta_i) + \frac{M^2}{R}\lambda j_0{}^2 R^\circ \frac{\varkappa_i}{\varkappa_e} = 0 \tag{2.3}$$

where

$$\varphi^2 = \gamma^\circ R^\circ (1 + \varkappa_i / \varkappa_e) \equiv \gamma L^2 (1 / \varkappa_e + 1 / \varkappa_i) \tag{2.4}$$

From (1.6) we have

$$\frac{\varkappa_e}{\varkappa_i} = \frac{13.58}{3.906}\left(\frac{m_i}{2m_e}\right)^{1/2}\left(\frac{T_e}{T_i}\right)^{5/2} \tag{2.5}$$

In actual cases $\varkappa_i / \varkappa_e \ll 1$, hence φ^2 can be written

$$\varphi^2 \approx \gamma^\circ R^\circ \equiv \gamma L^2 / \varkappa_i$$

The physical significance of φ^2 is quite clear. If $d^2\theta_i / d\eta^2 \sim \theta_e - \theta_i$ (see (2.2)), φ^2 expresses the ratio of energy transmitted from the electrons to ions during the elastic collisions, to that, lost by the ions through thermal conduction.

n	10^{16} cm^{-3}	10^{18} cm^{-3}
M^2	$2.7\cdot10^7$	$2.7\cdot10^7$
R	$1.5\cdot10^6$	$8.3\cdot10^7$
γ°	$1.8\cdot10^3$	$9.7\cdot10^4$
φ^2	$6.8\cdot10^8$	$2\cdot10^{12}$
$\delta = L/\varphi$ cm	$1.1\cdot10^{-3}$	$2.1\cdot10^{-5}$
σ cm^{-1}	$3.5\cdot10^{13}$	$6.4\cdot10^{13}$
$\varkappa_e/\varkappa_i$	$4.9\cdot10^3$	$4.9\cdot10^3$
λ	1.62	1.62

It is evident that the parameter φ^2 or an analogous one (e.g. $\gamma L^2 / \varkappa_e$) appears always when the energy equations of a two-temperature plasma is written in dimensionless form. Solutions of actual problems where this parameter is computed, are discussed in [3].

Let us compute the values of some magnitudes which we shall require for definite values of parameters, in case of a fully ionised plasma. Let $T_e = 1$ *ev*, $T_i = 1/4$ *ev*, $L = 30$ cm, $U = 10^5$ cm/sec, $m_i = 6.49 10^{-23}$ gm (atomic weight 39.1), $H = 10^3$ gauss and $n_e = n_n = n$. Results are given in the adjoining table. We see that $\varphi^2 \gg 1$. Hence the first term in Equation (2.3) will be of the same order as the second only when the gradient of the temperature difference is very great, i.e. if there is a layer adhering to the wall through which the temperature changes very sharply in the direction normal to the wall surface. We denote the thickness of such a layer by δ, and in the following we shall refer to it as the δ-layer.

Let us now write the orders of magnitude of the first and second term in Equation (2.3) within that layer

$$\frac{|(\theta_e - \theta_i)^\delta - (\theta_e - \theta_i)^w|}{\delta^2 / L^2} \approx \varphi^2 |\theta_e - \theta_i| \tag{2.6}$$

Here and in the following we shall use the suffix w to denote the quantities close to the wall, δ to denote the values at the distance δ from the wall, and ∞ to denote the values outside the wall, at infinity.

The quantity $|(\theta_e - \theta_i)^\delta - (\theta_e - \theta_i)^w| \sim |\theta_e - \theta_i|$ (i.e. the difference of temperature change in a layer of thickness δ close to the wall which would be of the same order as the difference itself) can only prevail in a layer the thickness of which is $\delta \sim L / \varphi \sim L / \sqrt{\gamma^\circ R^\circ} \equiv \sqrt{\varkappa_i / \gamma}$. Within the layer whose thickness is δ, the term $\partial^2 (\theta_e - \theta_i) / \partial \eta^2$ should be taken into account in Equation (2.3) while outside this layer the term can be ignored. Also, from (2.3) it follows that outside the layer the temperature difference is

$$\theta_e - \theta_i = (\theta_e - \theta_i)^\infty \approx \frac{M^2}{R} \lambda j_0^2 \frac{\varkappa_i}{\varkappa_e} \frac{1}{\gamma^\circ} \tag{2.7}$$

and, that depending on the current density and other parameters it can assume any values. Let us consider two particular cases in more detail.

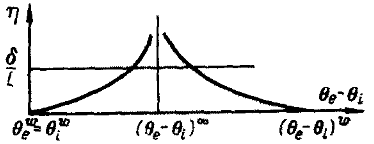

FIG. 1

1. Let $(\theta_e - \theta_i)^w \gg (\theta_e - \theta_i)^\delta$, i.e. the temperature difference close to the wall exceeds that at the distance δ from the wall. Temperature difference $\theta_e - \theta_i$, entering the right-hand side of Expression (2.6) will remain at the same order of magnitude as the difference of temperature close to the wall $(\theta_e - \theta_i)^w$ only within the layer of thickness $\delta \sim L / \sqrt{\gamma^\circ R^\circ} \equiv \sqrt{\varkappa_i / \gamma}$. Outside this layer the temperature difference between electrons and ions is of the same order as the temperature difference at infinity. (Formula (2.7)).

2. Let $(\theta_e - \theta_i)^\delta \gg (\theta_e - \theta_i)^w$ (for instance near the wall $\theta_e^w = \theta_i^w$, hence $(\theta_e - \theta_i)^w = 0$). We shall show, in which layer the temperature difference varies from zero at the wall, to $(\theta_e - \theta_i)^\infty$. Assuming that in the right-hand side of (2.6) $\theta_e - \theta_i = (\theta_e - \theta_i)$ we conclude that the change again takes place within a layer of thickness $\delta \sim L / \sqrt{\gamma^\circ R^\circ}$.

From this it follows that the difference in temperature of the electrons and ions occurring near the wall becomes equal to the temperature difference at ∞, within a thin layer of thickness δ.

Fig. 1 shows a qualitative picture of the possible behavior of the temperature difference between the electrons and ions in a layer of thickness $\delta \sim \sqrt{\varkappa_i / \gamma}$ in a quiescent plasma. The right-hand curve depicts the case when there is a large temperature difference close to the wall, so that $(\theta_e - \theta_i)^w \gg (\theta_e - \theta_i)^\infty$. The left hand curve illustrates the case where $\theta_e^w = \theta_i^w$. It is evident that in both cases the temperature difference close to the wall tends to that at infinity.

We thus see that in a two-temperature plasma a special kind of boundary layer of thickness $\delta \sim L / \sqrt{\gamma^\circ R^\circ} \equiv \sqrt{\varkappa_i / \gamma}$, can exist, within which a sharp change in temperature difference between electrons and ions occurs, ranging from the value of the difference at the wall to that at ∞. Let us study the way in which the electron and ion temperatures change within the δ-layer.

We shall solve Equations (2.1) and (2.2) assuming the transfer coefficients to be constant, for two types of boundary conditions

Case A when $\eta = \pm 1,\ \theta_e = \theta_e^{\,w},\ \theta_i = \theta_i^{\,w}$

$$\theta_e = \frac{1}{2}\,\alpha\varphi^2\theta^\infty(1-\eta^2) + \beta\,(\theta^w - \theta^\infty)\frac{\cosh\varphi y}{\cosh\varphi} + \beta\theta^\infty + \alpha\theta_e^{\,\infty} + \beta\theta_i^{\,w} \tag{2.8}$$

$$\theta_i = \frac{1}{2}\,\alpha\varphi^2\theta^\infty(1-\eta^2) - \alpha\,(\theta^w - \theta^\infty)\frac{\cosh\varphi y}{\cosh\varphi} - \alpha\theta^\infty + \alpha\theta_e^{\,w} + \beta\theta_i^{\,w}$$

Case B when $\eta = \pm 1,\ \theta_i = \theta_i^{\,w},\ \partial\theta_e / \partial\eta = 0$

$$\theta_e = \frac{1}{2}\,\alpha\varphi^2\theta^\infty(1-\eta^2) + \frac{\alpha\varphi\theta^\infty}{\beta\sinh\varphi}(\alpha\cosh\varphi + \beta\cosh\varphi y) + \theta^\infty + \theta_i^{\,w}$$

$$\theta_i = \frac{1}{2}\,\alpha\varphi^2\theta^\infty(1-\eta^2) + \frac{\alpha^2\varphi\theta^\infty}{\beta\sinh\varphi}(\cosh\varphi - \mathrm{ch}\,\varphi y) + \theta_i^{\,w}$$

$$\theta^\infty = (\theta_e - \theta_i)^\infty = \frac{M^2}{R}\lambda j_0^{\,2}\beta\,\frac{1}{\gamma^\circ}, \qquad \theta^w = \theta_e^{\,w} - \theta_i^{\,w} \tag{2.9}$$

$$\alpha = \left(1 + \frac{\varkappa_i}{\varkappa_e}\right)^{-1}, \qquad \beta = \frac{\varkappa_i}{\varkappa_e}\left(1 + \frac{\varkappa_i}{\varkappa_e}\right)^{-1}, \qquad \alpha + \beta = 1$$

Analysis of exact solutions gives a result coinciding with that obtained from the above qualitative considerations, namely that the difference of temperature of electrons and ions changes, within the δ-layer, from the values prevailing close to the wall, to the wall, to the value, $(\theta_e - \theta_i)^\infty$, constant in the main flow. When $\theta_e^{\,w} \gg \theta_i^{\,w}$, ion temperature increases with distance from the wall, electron temperature outside the δ-layer also increases, although it may diminish near the wall. Thus in the case A the electron temperature will decrease with distance from the wall if $(\theta_e - \theta_i)^w > (\theta_e - \theta_i)^\infty\,\varphi\varkappa_e/\varkappa_i$, and will increase when the inequality carries the opposite sign. Since $\varphi\varkappa_e/\varkappa_i \gg 1$, the temperature of electrons in the δ-layer will increase when the temperature differences close to the wall are not too great. We shall now consider two cases.

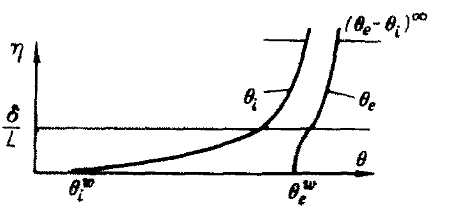

FIG. 2

1. Let $(\theta_e - \theta_i)^w \gg (\theta_e - \theta_i)^\infty$. Notice that in case B, the temperature difference developing on the wall satisfies this inequality. The solutions show clearly, that the ion temperature changes in the δ-layer exceed electron temperature changes by the factor $(\theta_e - \theta_i)^w$. Therefore the temperature difference close to the wall tends to the temperature difference at infinity in the δ-layer, and this, in general, results from the ion temperature change within the layer.

In fig. 2 we show a qualitative picture of the behavior of ion and electron temperatures in a quiescent plasma in the case when $(\theta_e - \theta_i)^w \gg (\theta_e - \theta_i)^\infty$. Electron temperature varies only slightly within a layer of thickness $\delta \sim \sqrt{\varkappa_i/\gamma}$ while ion temperature variation is relatively large. Therefore, the temperature difference between the electrons and ions tend to the difference at infinity in the δ-layer, and this is mainly due to the sharp changes in ion temperature.

Outside the δ-layer the electron and the ion temperature profiles change in a similar fashion, so that we always have the difference $\theta_e - \theta_i \sim (\theta_e - \theta_i)^\infty$.

2. Let the temperature difference close to the wall be small so that the difference $(\theta_e - \theta_i)^w \ll (\theta_e - \theta_i)^\infty$ (for example we can have $\theta_e^w = \theta_i^w$). From the analysis of Equation (2.8) it follows that the electron and the ion temperatures vary within the δ-layer by the same order of magnitude, inferring a temperature difference of electrons and ions outside the layer as $(\theta_e - \theta_i)^\infty$.

From this follows that the given temperature difference near the wall tends to the constant temperature difference prevailing at some distance away from the wall (at infinity), within a thin layer of thickness $\delta \sim L / \sqrt{\gamma^\circ R^\circ} \equiv \sqrt{\varkappa_i / \gamma}$. Also in this layer the ion temperature can vary by a magnitude appreciably exceeding the electron temperature variations.

Equations (2.1) and (2.2) describe the behavior of electrons and ions in a quiescent plasma in one dimension. However the results obtained will still be valid for a three-dimensional case. And indeed, although the terms $\partial^2 (\theta_e - \theta_i) / \partial\xi^2$, $\partial^2 (\theta_e - \theta_i) / \partial\zeta^2$, will appear in (2.3), because of the inequality $\varphi^2 \gg 1$, they can be neglected (this can be done of course, in the case, when we know in advance that there will be no sharp temperature change in the x- and z- direction).

The term $\partial^2 (\theta_e - \theta_i) / \partial\eta^2$ will remain, because sharp variation in temperature difference is expected in the direction normal to the wall. Thus Equation (2.3) remains unaltered. The derivatives of the ion and electron temperatures with respect to x and z will also enter (2.1) and (2.2). They will not however in general alter the qualitative structure of the behavior of the ion and electron temperatures in a thin layer of thickness δ.

In like manner, terms representing the work of viscous forces will not affect qualitatively, the temperature changes within the δ-layer.

3. The boundary layer in the case when there is temperature difference between electrons and ions in a moving plasma. Let us write the energy equations for the ions and electrons (1.3), (1.4) and (1.6) in dimensionless form. From the inequality $\omega_e\tau_e \ll 1$, together with the identity (1.7) it appears, that $|\mathbf{v}_e - \mathbf{v}_i| / U \ll 1$. (For clarity it is assumed that $j \sim \sigma UH / c$, so that $j_0 \sim 1$). The latter inequality allows considerable simplification of Equations (1.3) and (1.4). For instance instead of d_kT_k / dt we can write dT_k / dt, tensors of viscous stresses can be expressed as the derivatives of the average velocity of the mixture $\mathbf{v}$ instead of the derivatives of the velocities of each component. For clarity we will regard the flow steady, and the medium incompressible. Extension to the case of compressible fluid is easy.

With the above assumptions, Equations (1.3) and (1.4) become

$$\frac{3}{2} u \frac{\partial\theta_e}{\partial\xi} + \frac{3}{2} v \frac{\partial\theta_e}{\partial\xi} = \frac{\partial}{\partial\xi} \frac{1}{R^\circ\varkappa_i/\varkappa_e} \frac{\partial\theta_e}{\partial\xi} + \frac{\partial}{\partial\eta} \frac{1}{R^\circ\varkappa_i/\varkappa_e} \frac{\partial\theta_e}{\partial\eta} + \\ + \frac{M^2}{R} \lambda j_0^2 + \frac{\eta_e}{\eta_i} \frac{\lambda}{R} \Phi - \gamma^\circ (\theta_e - \theta_i) \tag{3.1}$$

$$\frac{3}{2}u\frac{\partial\theta_i}{\partial\xi}+\frac{3}{2}v\frac{\partial\theta_i}{\partial\eta}=\frac{\partial}{\partial\xi}\frac{1}{R^\circ}\frac{\partial\theta_i}{\partial\xi}+\frac{\partial}{\partial\eta}\frac{1}{R^\circ}\frac{\partial\theta_i}{\partial\eta}+\frac{\lambda}{R}\Phi+\gamma^\circ(\theta_e-\theta_i)$$
$$\left(\Phi=2\left[\left(\frac{\partial u}{\partial\xi}\right)^2+\left(\frac{\partial v}{\partial\eta}\right)^2\right]+\left(\frac{\partial v}{\partial\xi}+\frac{\partial u}{\partial\eta}\right)^2\right) \tag{3.2}$$

For convenience we shall denote the terms in Equations (3.1) and (3.2), by the letters $W_k{}^e$, $W_k{}^i$, where k is the number of the term. The order of magnitude of term W_5^e, describing Joule heating, is $M^2\lambda j_0{}^2/R\sim\lambda$. Term W_6^e, describing the work against the forces of electron viscosity can be of the same order as other terms entering (3.1), for instance Joule heating within and only within the dynamic, viscous boundary layer; outside this layer $\partial u/\partial\eta=0$ and $W_6^e=0$. It can however be shown, that when $T_e\approx T_i$ and when inside the viscous layer, the term describing the work against electronic viscosity is much smaller than the Joule heating. Actually if we compare the order of magnitude of the term corresponding to work of electron viscous forces and that of the Joule heating within the viscous boundary layer, we obtain

$$\frac{W_6{}^e}{W_5{}^e}\approx\frac{\eta_e}{\eta_i}\approx\left(\frac{T_e}{T_i}\right)^{5/2}\left(\frac{m_e}{2m_i}\right)^{1/2} \tag{3.3}$$

In the following, for definiteness, we shall consider the values of parameters for which the term representing the work of electron viscous forces is small and can be neglected in (3.1). When the electron temperature increases, the work against electron viscous forces increases too. Evidently cases can be envisaged where ratio (3.3) becomes greater than, or equal to unity, and the term in question should then be included in (3.1).

We will now turn to the behavior of the temperature difference $\theta_e-\theta_i$ in the immediate neighbourhood of the wall (within the ionic temperature boundary layer, see section 4). For convenience we will assume R° and $R^\circ\varkappa_i/\varkappa_e$ constant, bearing in mind that we are considering the behavior of the temperature in the boundary layers close to the wall. We shall further neglect the second derivatives of temperature with respect to η as compared with the derivatives of η.

Let us subtract (3.2) multiplied by R° from (3.1) multiplied by $R^\circ\varkappa_i/\varkappa_e$. Taking into account the previous estimates we arrive at the following (in section 4 it was shown that in the number of cases, the term W_2^e can be neglected in (3.1))

$$\frac{3}{2}u\frac{\partial\theta_e}{\partial\xi}\frac{\varkappa_i}{\varkappa_e}R^\circ+\frac{3}{2}v\frac{\partial\theta_e}{\partial\eta}\frac{\varkappa_i}{\varkappa_e}R^\circ-\frac{3}{2}u\frac{\partial\theta_i}{\partial\xi}R^\circ-\frac{3}{2}v\frac{\partial\theta_i}{\partial\eta}R^\circ=$$
$$=\frac{\partial^2(\theta_e-\theta_i)}{\partial\eta^2}-\varphi^2(\theta_e-\theta_i)-\lambda\left(\frac{\partial u}{\partial\eta}\right)^2+\frac{M^2}{R}\lambda j_0{}^2R^\circ\frac{\varkappa_i}{\varkappa_e} \tag{3.4}$$

As before φ^q is determined from Formula (2.4).

Now we shall estimate the order of magnitude of the terms which enter (3.4). In many actual cases, as we have already shown, $\varphi^2\gg 1$. Values of φ^2 are given in the table. The first term on the right-hand side which is connected with thermal conductivity, can be of the same order as the second term on the right-hand side only, if the gradient of the temperature difference is very great, i.e. if, close to the wall there is a layer in which a rapid temperature change occurs in the direction normal to the wall. Let us estimate the order of magnitude of the thickness of this layer, denoting it by δ. To do this we shall write the orders of the first and second terms on the right-hand sides of Equation (3.4)

$$\frac{|(\theta_e - \theta_i)^\delta - (\theta_e - \theta_i)^w|}{\delta^2/L^2}, \qquad \varphi^2 |\theta_e - \theta_i| \tag{3.5}$$

From this it follows that when $|(\theta_e - \theta_i)^\delta - (\theta_e - \theta_i)^w| \sim |\theta_e - \theta_i|$ the terms considered will all be of the same order when $\delta \sim L/\varphi \approx \sqrt{\varkappa_i / \gamma}$. This δ-layer will be equivalent to the δ-layer discussed in section 2 in connection with the quiescent plasma. Within the δ-layer the first term on the right-hand side of (3.4) should be taken into account, outside the layer however, it can be neglected. Note that when $\lambda / \gamma^\circ \sim 1$ outside the δ-layer ($\delta \sim \delta_i$ when $\lambda / \gamma^\circ \sim 1$, section 4), and when $\lambda / \gamma^\circ \gg 1$ outside the δ_i-layer, Equation (3.4) transforms into an equation for the electron temperature (3.1), so that an increase in temperature difference results from the increase in electron temperature for very slight change in the ion temperature.

The thickness of the δ-layer is $\sqrt{\gamma^\circ}$ times less that the thickness of the viscous boundary layer (section 4). When $\sqrt{\gamma^\circ} \sim 1$ the thickness of the δ-layer is of the order of δ_v which is the thickness of the viscous boundary layer. At the same time, within the δ-layer the order of the term $(\partial u / \partial \eta)^2 \sim R$. Evidently the order of this term as well as that of the derivative $\partial \theta_i / \partial \eta$ remains as before within the δ-layer, and in case when $\delta \ll \delta_v$ (for $\gamma^\circ \gg 1$). When $\gamma^\circ \gg 1$, the component of velocity u has no time to react in time the value u^∞ within the δ-layer. The order of $u^\delta \approx u^\infty / \sqrt{\gamma^\circ}$. It follows from the continuity equation that the orders of terms $\partial u / \partial \xi$ and $\partial v / \partial \eta$ coincide, hence within the δ-layer we have

$$v \sim \frac{\partial u}{\partial \xi} \frac{\delta}{L} \sim \frac{1}{\sqrt{\gamma^\circ}} \frac{1}{\sqrt{\gamma^\circ R^\circ}} \sim \frac{1}{\gamma^\circ \sqrt{R^\circ}}$$

Evidently the order of magnitude of the derivatives $\partial \theta_e / \partial \xi$, and $\partial \theta_i / \partial \xi$ also decreases within the δ-layer although it can be shown, that the terms which contain these derivatives can be neglected, when the derivatives ore of the order of unity. Indeed, using the previous estimates we can easily show that the third term on the left-hand side of (3.4) is $\lambda \sqrt{\gamma^\circ}$ times smaller than the third viscous term in the right-hand side of (3.4) and, it can be neglected when $\lambda \sqrt{\gamma^\circ} \gg 1$. Similarly the first term on the left-hand side is $\lambda \sqrt{\gamma^\circ}$ times smaller than the last term on the right-hand side, and can also be neglected when $\lambda \sqrt{\gamma^\circ} \gg 1$. The last term on the left-hand side is $\gamma^\circ \sqrt{\lambda}$ times smaller than the viscous term (third term on the right-hand side). The last term on the right-hand side is $\varkappa_i / \varkappa_e$ times smaller than the penultimate one, and can also be neglected. With use of these estimates, equation (3.4) can be simplified. When $\lambda \sim 1$, and $\gamma^\circ \gg 1$ Equation (3.4) assumes a simple form

$$\frac{\partial^2 (\theta_e - \theta_i)}{\partial \eta^2} - \varphi^2 (\theta_e - \theta_i) - \lambda \left(\frac{\partial u}{\partial \eta}\right)^2 = 0 \tag{3.6}$$

When $\gamma^\circ \gg 1$, convection terms are absent from the equation for the temperature difference in the δ-layer, since the velocity within this layer is very small.

It has already been mentioned that outside the δ-layer the second derivative of temperature difference cannot be neglected. Using (3.6) it is easy to write the order of the temperature difference on the outer boundary of the δ-layer

$$(\theta_e - \theta_i)^\delta \equiv - \frac{\lambda}{\gamma^\circ R^\circ} \left(\frac{\partial u}{\partial \eta}\right)^2 \approx - \frac{\lambda}{\gamma^\circ} \; (\gamma^\circ \gg 1) \tag{3.7}$$

For the values of $\gamma^{\circ} \sim 1$, the temperature difference is described by a differential equation which can easily be obtained from (3.4), using the above estimates. Nevertheless, the order of the temperature difference at the outer boundary of the δ-layer is still $(\theta_e - \theta_i)^{\delta} \approx -\lambda$. The negative sign in (3.7) results from the fact, that in the δ-layer the ions heat up to a greater degree than the electrons. This statement is connected with the assumption that $M^2 / R \sim 1$, $j_0 \sim 1$, and $\lambda\,(\partial u / \partial\eta)^2 \sim \lambda R$; of course, when $M^2 / R \gg 1$ or $j_0 \gg 1$ or $(\partial u / \partial\eta)^2 \not\approx R$, Joule heating in the δ-layer can exceed the viscous dissipation, so that the temperature of the electrons will exceed that of the ions.

In section 5 we shall show that in the outside stream the temperature difference is

$$(\theta_e - \theta_i) \approx (\theta_e - \theta_i)^{\infty} = \frac{M^2}{R}\lambda j_0{}^2 \frac{1}{2\gamma^{\circ}}, \quad \theta_e - \theta_i \sim \frac{\lambda}{\gamma^{\circ}} \qquad \left(\text{when} \frac{M^2}{R} \sim 1\right)$$

Thus, whether the temperature difference close to the wall is $\theta^{w}{}_e = \theta_i{}^{w}$ or $(\theta_e - \theta_i)^{w} \gg (\theta_e - \theta_i)^{\infty}$, in a layer of thickness δ, the temperature difference, in general, varies from the value close to the wall $(\theta_e - \theta_i)^{w}$ to the value, the modulus of which is of the order smaller or equal to the value $(\theta_e - \theta_i)^{\infty}$ at infinity. This is also valid for values $\gamma^{\circ} \sim 1$, $R^{\circ} \gg 1$.

In other words, whatever the temperature difference existing close to the wall, at a distance $\delta \sim L / \sqrt{\gamma^{\circ} R^{\circ}} \equiv \sqrt{\varkappa_i / \gamma}$ from the wall, screening of that temperature difference occurs. Also, at the outer boundary of the δ-layer, the temperature difference reaches the values which are defined by the collateral action of Joule heating, viscous heating and thermal conductivity when $\gamma^{\circ} \gg 1$, and with the convection terms added, when $\gamma^{\circ} \sim 1$. Let us examine the behavior of electron and ion temperatures within the δ-layer, using Equations (3.1) and (3.2).

It has already been shown that the term W_6^e can be neglected from the equation over a large range of parameter variation. Since we are interested in the phenomena within the temperature boundary layers, we will neglect the second temperature derivatives with respect to x. Besides, it was shown in section 4, that in the electron temperature boundary layer, the term W_2^e can be neglected in several cases. It is easy to see that in the δ-layer this term could have been ommitted from the outset.

If we carry out further evaluations analogous to those used to simplify Equation (3.4), we can show that the terms $W_1{}^e \sim 1/\sqrt{\gamma^{\circ}}$, $W_1{}^i \sim 1 / \sqrt{\gamma^{\circ}}$, and $W_2{}^i \sim 1 / \gamma^{\circ}$ will be smaller than (when $\gamma^{\circ} \gg 1$) and of the order (when $\gamma^{\circ} \sim 1$) of the terms $W_5{}^e \sim \lambda$, and $W_5{}^i \sim \lambda$ $(\lambda \sim 1)$ respectively. In the case when $\gamma^{\circ} \gg 1$, terms $W_1{}^e$, $W_1{}^i$, and W_2^i can be neglected, and Equations (3.1) and (3.4) assume, within the δ-layer, a form similar to that of (2.1) and (2.2); it is only because of the influence of the term of viscous dissipation in the equation for the ion component, that the difference of the temperatures of electrons and ions at the outer boundary of the δ-layer is of the order $(-\lambda/\gamma^{\circ})$ and $\varkappa_e / \varkappa_i$ times greater in terms of the absolute magnitudes, than the temperature differences worked out in the problem discussed in section 2

$$\frac{\partial^2\theta_e}{\partial\eta^2} + \frac{M^2}{R}\lambda j_0{}^2 R^{\circ} \frac{\varkappa_i}{\varkappa_e} - \gamma^{\circ} R^{\circ} \frac{\varkappa_i}{\varkappa_e}(\theta_e - \theta_i) = 0 \tag{3.8}$$

$$\frac{\partial^2\theta_i}{\partial\eta^2}+\lambda\left(\frac{\partial u}{\partial\eta}\right)^2+\gamma^\circ R^\circ(\theta_e-\theta_i)=0$$

Although these equations are easy to solve, the required result can easily be obtained from a qualitative investigation.

The difference of the temperatures of the electrons and ions entering Equation (3.8), will be, near the wall, of the order of the difference $(\theta_e-\theta_i)^w$ in case 1, when $(\theta_e-\theta_i)^w \gg (\theta_e-\theta_i)^\delta$ and less than, or of the order $(\theta_e-\theta_i)^\delta$ in case 2, when $(\theta_e-\theta_i)^w \leqslant (\theta_e-\theta_i)^\delta$.

Using these estimates and Equations (3.8) we can write the orders of the second derivatives of the electron and ion temperatures.

In case 1

$$\frac{\partial^2\theta_e}{\partial\eta^2}\sim(\theta_e-\theta_i)^w\gamma^\circ R^\circ\ \frac{\varkappa_i}{\varkappa_e},\qquad \frac{\partial^2\theta_i}{\partial\eta^2}\sim-(\theta_e-\theta_i)^w\gamma^\circ R^\circ \tag{3.9}$$

In case 2

$$\frac{\partial^2\theta_e}{\partial\eta^2}\sim(\theta_e-\theta_i)^\delta\gamma^\circ R^\circ\frac{\varkappa_i}{\varkappa_e},\qquad \frac{\partial^2\theta_i}{\partial\eta^2}\sim+(\theta_e-\theta_i)^\delta\gamma^\circ R^\circ \tag{3.10}$$

Let us write the second derivatives of temperature in the δ-layer

$$\frac{\partial^2\theta_k}{\partial\eta^2}\sim\left[\left(\frac{\partial\theta_k}{\partial\eta}\right)^\delta-\left(\frac{\partial\theta_k}{\partial\eta}\right)^w\right]\frac{1}{\delta}\sim\frac{\theta_k{}^w-\theta_k{}^\delta}{1/\gamma^\circ R^\circ}-\frac{1}{\delta}\left(\frac{\partial\theta_k}{\partial\eta}\right)^\delta \tag{3.11}$$

Assuming that $(\partial\theta_e/\partial\eta)^\delta\sim(\partial\theta_i/\partial\eta)^\delta$, we can conclude from (3.8) to (3.11) that in both cases the ion temperature varies within the δ-layer by an amount which is much greater than the change in the temperature of the electrons, therefore the change in the given temperature differences between electrons and ions from the value near the wall to the value at the outer boundary of the δ-layer is caused mainly by the sharp change in the temperature of the ions. A possible temperature profile is shown in fig. 3 which is described in section 4. When $\gamma^\circ\sim1$, the thickness of the δ-layer coincides with the thickness of the viscous and the ionic temperature boundary layers. Using the estimates given above, we can easily write the equation analogous to (3.6) describing the behavior of the temperature differences in the δ-layer. It is also easy to perform similar estimates, when the order of the term $\lambda(\partial u/\partial\eta)^2$ within the δ-layer (when $\gamma^\circ\gg1$), differs from λR, and $j_0\gg1$.

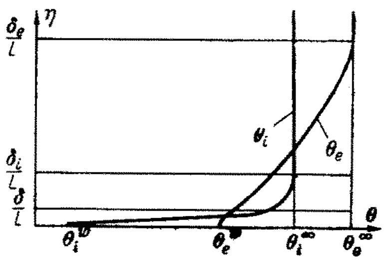

FIG. 3

4. Electron and ion temperature boundary layers. We will now discuss possible ways of simplifying Equations (3.1) and (3.2) in the temperature boundary layers when $R^\circ\gg1$ and $R^\circ\varkappa_i/\varkappa_e\gg1$. Note that since $\varkappa_i/\varkappa_e\ll1$. (Formula (2.5)), a situation can arise when $R^\circ\gg1$ and $R^\circ\varkappa_i/\varkappa_e\lesssim1$. This case will be discussed at the end of this section.

In section 5 we will show that the temperature difference in the outer stream is

$$\theta_e-\theta_i\sim M^2\lambda j_0{}^2/2R\gamma^\circ$$

However, a situation is possible when at the wall and therefore in its immediate proximity, or at the channel entry, the temperature difference is found, which is much greater than the temperature difference established in the outer stream. Hence a situation is possible when the terms W_7^e and W_6^i are of next higher order, than the terms W_5^e and W_5^i respectively. When we estimate the thickness of ion and electron temperature boundary layers we must compare the orders of magnitude of terms containing the second derivative in η with characteristic (maximum) order of the other terms.

It has been shown in the preceding paragraph that the large temperature difference established close to the wall evens out within the thin δ-layer of the thickness $\delta \sim L / \sqrt{\gamma^\circ R^\circ}$ (i.e. it tends to the values determined by Joule heating, viscosity and other terms appearing in the equations). Both, the thickness of the layer, and the temperature difference within it are known, therefore comparing the orders of the terms W_4^e and W_7^e with that of W_4^i, we can arrive at the order of the electron and ion temperatures at the outer boundary of the δ-layer, as was done in section 3. Terms associated with thermal conductivity (W_4^e and W_4^i) are significant not only within the δ-layer, but also at some distance outside it, within the temperature boundary layers where the term $\gamma^\circ(\theta_e - \theta_i)$ is smaller than or of the same order as the terms describing Joule or viscous dissipation.

Now let us estimate the order of magnitude of these distances. They will, in fact, be the thicknesses of the temperature boundary layers. Terms depending on electron thermal conductivity can be of the same order as the terms describing Joule heating only, when the gradient $\partial\theta_e / \partial\eta$ is very large, i.e. when there is a layer near the wall, in which a rapid change in electron temperature takes place in the direction normal to the wall. We will call such a layer the electron temperature boundary layer, and denote its thickness by δ_e.

We will assume that $\partial^2\theta_e / \partial\xi^2 \sim 1$, and that this term can be neglected in comparison with $\partial^2\theta_e / \partial\eta^2 \sim \delta_e^{-2}$. Comparing the orders of the terms W_4^e and W_5^e, we conclude that the transport of heat due to electron thermal conductivity will be of the same order as heat dissipated by Joule heating only, when the electron temperature boundary layer thickness satisfies the following

$$\left(\frac{\delta_e}{L}\right)^2 \sim \frac{R\varkappa_e}{M^2\lambda j_0^2 R^\circ \varkappa_i}, \qquad \left(\frac{\delta_e}{L}\right)^2 \sim \frac{\varkappa_e}{\varkappa_i}\frac{1}{\lambda R^\circ} \quad \text{when} \quad \frac{M^2}{R} \sim 1,\ j_0 \sim 1 \tag{4.1}$$

In section 1, we have estimated the quantities determining the viscous boundary layer. In this connection we should remember, that the thickness of the dynamic boundary layer is given by $\delta_v \sim L / \sqrt{R}$.

Let us now compare the order of the thickness δ_e with that of the viscous boundary layer, when $M^2 / R \sim 1$

$$\frac{\delta_v}{\delta_e} \sim \left(\lambda \frac{\varkappa_i}{\varkappa_e}\frac{R^\circ}{R}\right)^{1/2} \sim \left(0.246\,\lambda \frac{\varkappa_i}{\varkappa_e}\right)^{1/2} \tag{4.2}$$

The orders of the quantities $\varkappa_i / \varkappa_e$ and λ are shown in the table. It is clear that in many practical cases $\lambda \lesssim 1$, $M^2 / R \sim 1$. Thus in a fully ionised plasma $\delta_e \gg \delta_v$, i.e. the thickness of the electron temperature boundary layer is very much greater than that of the viscous boundary layer. In the previous section it was shown that the thickness of the δ-layer was less than (when $\sqrt{\gamma^\circ} \gg 1$), or of the same order as (when $\sqrt{\gamma^\circ} \sim 1$)) the thickness of the viscous boundary layer. Hence, from the inequality $\delta_e \gg \delta_v$ it follows, that $\delta_e \gg \delta$ is even more pronounced. Thus in practice, the thickness δ_e can be measured from the wall just as in the case when $W_7^e \sim W_5^e$. Generally speaking, the first term in Equation (3.1) is of the same order of magnitude as the term describing Joule heating. Outside the viscous boundary layer $v = 0$ and the term W_2^e can be neglected. The order of the term W_2^e within the viscous boundary layer is equal to δ_v / δ_e. Comparing the orders of magnitude of the terms W_2^e and W_5^e, we have

$$\frac{W_2^e}{W_5^e} \sim \sqrt{\varkappa_i / \lambda\varkappa_e} \qquad (M^2 / R \sim 1) \tag{4.3}$$

It can be seen from the table that this ratio is, in a fully ionised plasma, very much smaller that unity in many actual cases, therefore W_2^e in Equation (3.1) can be neglected.

Now let us estimate the orders of the terms in Equation (3.2). Terms describing the ionic thermal conductivity can be of the same order as those describing the viscous ionic dissipation, only in the case, when the gradient $\partial\theta_i / \partial\eta$ is very large, i.e. if there is a layer near the wall, exibiting a sharp temperature variation in the direction normal to that wall. This layer will be referred to as the ionic temperature boundary layer. Its thickness will be denoted by δ_i (remembering, when $W_6^i \gg W_5^i$, the thickness of the δ_i-layer is measured not from the wall, but from the outer boundary of the δ-layer). As in the case of the electron layer, we neglect the term $\partial^2\theta_i / \partial\xi^2$ as compared with $\partial^2\theta_i / \partial\eta^2$. Comparing the orders of the terms W_4^i and W_5^i, we come to the conclusion that heat transport due to the ionic thermal conductivity will be of same order as the heat evolved in the work against the forces of ionic viscosity only, if the thickness of the ionic boundary layer satisfies the following relation (remebering that $M^2 / R \sim 1$)

$$(\delta_i / L)^2 \sim 1 / \lambda R^\circ \tag{4.4}$$

Now let us compare the order of the quantity δ_i with the thickness of the viscous boundary layer

$$\delta_v / \delta_i \sim \sqrt{0.246}\,\lambda \sim {}^1/_2 \sqrt{\lambda} \tag{4.5}$$

Using the data from the table we find, that the thickness of the ionic temperature boundary layer, is of the same order as that of the viscous boundary layer. If we compare the thickness of the δ-layer with the thickness of the ionic temperature boundary layer $(M^2 / R \sim 1)$, we find that, when $\gamma^\circ / \lambda \gg 1$ the thickness $\delta_i \gg \delta$, , while when $\gamma^\circ / \lambda \sim 1$,the thickness $\delta_i \sim \delta$. We should also remember that in a highly ionised plasma $\gamma^\circ \gg 1$, and $\lambda \lesssim 1$, therefore when $W_6^i \gg W_5^i$,the thickness of the δ_i-layer can be measured from the wall. This can also be done when $\gamma^\circ \sim 1$.

Usually, the order of magnitude of the first term in Equation (3.2) is the same as that of the term representing viscous dissipation. The second term in (3.2) is of the order of the ratio δ_v / δ_i. If we compare the second term with the fifth we get

$$W_2^i / W_5^i \sim \sqrt{\lambda R^\circ / R} = \sqrt{0.246\lambda} \tag{4.6}$$

for the values of the parameter, for which this ratio is less than unity, and the second term in Equation (3.2), can be neglected.

The equations representing the electron and ionic temperature boundary layers take the following form

$$\frac{3}{2}u\frac{\partial\theta_e}{\partial\xi}+\frac{3}{2}v\frac{\partial\theta_e}{\partial\eta}=\frac{\partial}{\partial\eta}\frac{1}{R^\circ\varkappa_i/\varkappa_e}\frac{\partial\theta_e}{\partial\eta}+\frac{M^2}{R}\lambda j_0^2-\gamma^\circ(\theta_e-\theta_i) \qquad (4.7)$$

$$\frac{3}{2}u\frac{\partial\theta_i}{\partial\xi}+\frac{3}{2}v\frac{\partial\theta_i}{\partial\eta}=\frac{\partial}{\partial\eta}\frac{1}{R^\circ}\frac{\partial\theta_i}{\partial\eta}+\frac{\lambda}{R}\left(\frac{\partial u}{\partial\eta}\right)^2+\gamma^\circ(\theta_e-\theta_i) \qquad (4.8)$$

Equation (4.7) represents the case, when the ratio $W_2^e / W_5^e \sim 1$ Formula (4.3)).

Now let us compare the thicknesses of the electron and ionic temperature layers

$$\delta_i / \delta_e \sim \sqrt{\varkappa_i / \varkappa_e} \qquad (4.9)$$

It is clear from the table, that in a fully ionised plasma where $\varkappa_i / \varkappa_e \ll 1$, the electron temperature boundary layer is much thicker than the ionic temperature boundary layer. We should note that increase in the ratio θ_e / θ_i leads to the increase in the ratio δ_e / δ_i while when M^2/R increases the thicknesses of the electron and ionic temperature layers decrease.

Figure 3 gives a qualitative picture of the possible behavior of the ion and electron temperatures in a moving plasma, when $\gamma^\circ \gg 1$ (under these conditions $\delta \ll \delta_i \ll \delta_e$) and $\theta_e^w \gg \theta_i^w$. The electron and ion temperatures vary from the values at the wall to the values prevailing in the outer flow, over the electron (of thickness δ_e) and ionic (of thickness δ_i) temperature boundary layers respectively. In a layer of thickness δ a sharp change in the ionic temperature takes place, from the value close to the wall to the value determined by ionic viscous dissipation in accordance with the equations given above.

Outside the δ-layer but within the ionic temperature layer, the ionic temperature will increase; the temperature difference is described by Equation (3.4) in which the term with the second order derivative can be neglected. Outside the ionic temperature layer, Equation (3.4) transforms into (3.1) and the temperature difference varies only at the expense of temperatures of the electrons with the ionic temperature remaining constant, while the ion temperature obeys the equations for the external stream. It should be noted, that the magnitudes of the ratios $\delta_e/\delta_i = \sqrt{\varkappa_e / \varkappa_i} \gg 1$, $\delta_i / \delta \gg 1$ (see the table and fig. 3) are considerably reduced.

If $R^\circ \gg 1$ while $R^\circ\varkappa_i / \varkappa_e \lesssim 1$, then the ionic temperature boundary layer exists, and as before, is described by Equation (4.8) while the electron temperature boundary layer is absent. The electron temperature should then be determined from Equation (3.1) in which the work against forces of electron viscosity (estimate (3.3)), and the second term (estimate (4.3)) can, in most cases, be neglected. Equation (3.1) for electron temperatures (elliptical) becomes then

$$\begin{aligned}\frac{3}{2}u\frac{\partial\theta_e}{\partial\xi}+\frac{3}{2}v\frac{\partial\theta_e}{\partial\eta}&=\frac{\partial}{\partial\xi}\frac{1}{R^\circ\varkappa_i/\varkappa_e}\frac{\partial\theta_e}{\partial\xi}+\\&+\frac{\partial}{\partial\eta}\frac{1}{R^\circ\varkappa_i/\varkappa_e}\frac{\partial\theta_e}{\partial\eta}+\frac{M^2}{R}\lambda j_0^2-\gamma^\circ(\theta_e-\theta_i)\end{aligned} \qquad (4.10)$$

5. Equations for electron and ionic temperatures in the external stream. Let us write the equations for electron and ionic temperatures in the external stream in the form

$$\frac{3}{2}u\frac{\partial\theta_e}{\partial\xi}=\frac{M^2}{R}\lambda j_0^2-\gamma^\circ(\theta_e-\theta_i) \qquad (5.1)$$

$$\frac{3}{2} u \partial \theta_i / \partial \xi = \gamma^\circ (\theta_e - \theta_i) \tag{5.2}$$

From the table it is evident that $\gamma^\circ \gg 1$, over a wide range of variation of the parameters. The large values of γ° define the specific behavior of electron and ion temperature differences $\theta_e - \theta_i$ along the channel. Subtracting (5.2) from (5.1), we obtain

$$\frac{3}{2} u \frac{\partial (\theta_e - \theta_i)}{\partial \xi} = \frac{M^2}{R} \lambda j_0^2 - 2\gamma^\circ (\theta_e - \theta_i) \tag{5.3}$$

The term on the left-hand side of (5.3) can be of the same order as the second term on the right-hand side only within a thin layer, which will be referred to as the γ-layer, and which has a large temperature gradient. Outside this γ-layer

$$\theta_e - \theta_i = (\theta_e - \theta_i)^\infty = \frac{M^2}{R} \lambda j_0^2 \frac{1}{2\gamma^\circ}$$

A. Suppose that the difference $(\theta_e - \theta_i)_1 \gg (\theta_e - \theta_i)^\infty$. is given at the cross-section $\xi = \xi_1$. It is easily seen that the difference $(\theta_e - \theta_i)_1$ tends to the value $(\theta_e - \theta_i)^\infty$ in a thin γ-layer of thickness $l \sim 3Lu / 4\gamma^\circ$ while outside this layer we have, $\theta_e - \theta_i = (\theta_e - \theta_i)^\infty$.

B. Now suppose that the difference

$(\theta_e - \theta_i)_1 \ll (\theta_e - \theta_i)^\infty$ is given at the cross-section $\xi = \xi_1$ (for instancewe could have $(\theta_e - \theta_i)_1 = 0$). The difference $\theta_e - \theta_i$ tends to the value $(\theta_e - \theta_i)^\infty$ within the thin γ-layer of the same thickness $l \sim 3Lu / 4\gamma^\circ$.

It is also possible to predict the qualitative behavior of the temperature profiles θ_e and θ_i within and without the γ-layer in the manner analogous to that in section 2. when we studied the behavior of the temperatures close to the wall in a quiescent plasma.

It should be noted that at large values of γ° in the γ-layer, the temperatures of the ions and electrons can vary rapidly. In general therefore, such values of the parameters are possible, for which the thermal conductivity of the components should be taken into account, when determining the temperature profiles of electrons and ions inside the γ-layer. We should also note that, when $u \sim \gamma^\circ$, $\delta^*/L \sim 1$. When this happens, the plasma flowing through the channel passes through it so rapidly, that the relaxation of electron and ion temperature takes place along the whole length of the channel. When $u \gg \gamma^\circ$ the plasma travels so rapidly, that in practice, the relaxation does not have time to take place.

It should be pointed out that the presence of walls is not an essential requirement either in the formation of the δ-layer nor in the case of the γ-layer in the external stream. Any temperature difference caused by the outside sources at any channel section will attain the values, which are determined, generally speaking by the combined effect of Joule heating, viscosity, convection and thermal conductivity within the thin layer of thickness δ in agreement with what was said in section 3.

Note also the assumption made about the incompressibility of the medium in the present studies is not in itself significant,and was only done to cut down the length of the equations.

6. Method of approximate estimation of the temperature boundary layers. We can use the above estimates of the thickness of the boundary layers for a suggested approximate method of solving the boundary layer equations. It should be noted that, in general, even in the case of incompressible fluid, the dynamic problem is not divorced from the thermal one. This is because the coefficient of transport depends on the electron and ion temperatures. Let us assume that the problem is solved for the external stream. From section 4, it follows, that $\delta_e \gg \delta_i \sim \delta_v$. Within the δ_i-layer the ion temperature satisfies (4.8). Outside the δ_i-layer the ion temperature satisfies (5.2) for the external stream. Everywhere within the δ_e-layer and also outside the δ_i-layer, the electron temperature satisfies Equation (4.7) for the electron temperature boundary layer.

Let us work out the electron temperature distribution in the first approximation.

Into Equation (4.7) for electron temperature let us insert the values of all the parameters equal to those in the external stream, i.e. velocity equal to that in the outside stream, likewise ion temperatures, etc.

Knowing the distribution of all the parameters in the outside stream it is possible to find the temperature distribution of the electrons in the electron boundary layer, from the equation obtained. This distribution will be, in general, close to the real one everywhere with the exception of a region close to the wall and of thickness of the order of the ion temperature layer. In order to find the ion temperature distribution in the ion boundary layer and the velocity distribution in the viscous boundary layer, we must insert into Equation (4.8) for ion temperature and into (1.2) for the equations of motion, the electron temperature found in the approximate method described above. Note here that the electron temperature not only enters the relaxation term $\gamma^{\circ} (\theta_e - \theta_i)$, but also the transport coefficient. Solving these equations together with equations of continuity, of state, and the Maxwell equations we can, to a first approximation find the distribution of velocity components, of the ion temperature, the density and other quantities within a layer of thickness $\delta_i \sim \delta_v$.

Improved values of electron and ion temperatures can be obtained by subsequent approximations. Thus to obtain the electron temperature distribution in a second approximation, values from the first approximation of ion temperatures, velocity components, etc. obtained from the solutions of the viscous boundary layer problem, should be inserted in (4.7). After that, the solution of the equation for the electron temperature gives us the second approximation to the electron temperature distribution in the electron boundary layer.

Using the second approximation for the electron temperature it is possible to construct, by the same method, the second approximation for the ion temperatures, velocities etc.

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